

# Shadow removal

When taking a picture, what color is a (Lambertian) surface?



What if it's not a cloudy day?



Region lit by  
skylight only

Region lit by  
sunlight and  
skylight

What great things could we do if  
we could easily find shadows?











# An Intrinsic Image

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- What effect is the **lighting** having, irrespective of surface materials?
- What is the **surface reflectance**, irrespective of lighting?

Tappen et al. PAMI'05



Original



Lighting/Shading

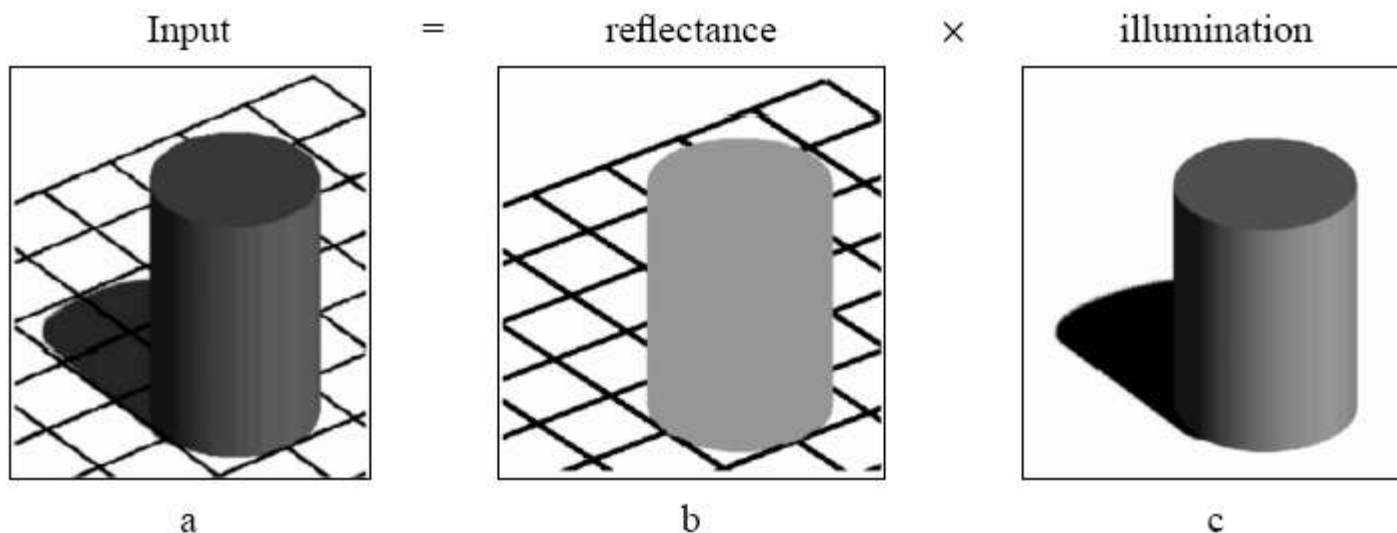


Reflectance

# Pursuit of Intrinsic Images (1)

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- Lightness and Retinex Theory
  - Land & McCann '71
- Recovering Intrinsic Scene Characteristics From Images
  - Barrow & Tenenbaum '78



# Pursuit of Intrinsic Images (2)

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- Painted Polyhedra - ICCV'93
- Image Sequences - ICCV'01
- Single Image - NIPS'03
- Entropy Minimization - ECCV'04

# Pursuit of Intrinsic Images (2)

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- Painted Polyhedra - ICCV'93 (Generative)
- Image Sequences - ICCV'01 (Discriminative)
- Single Image - NIPS'03 (Discriminative)
- Entropy Minimization - ECCV'04 (Generative)

# Image Sequences

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- Deriving Intrinsic Images from Image Sequences
  - Weiss ICCV'01
- For static objects, **multiple frames**



a



b



c

# Problem Formulation

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Given a sequence of  $T$  images  $\{I(x, y, t)\}_{t=1}^T$

in which reflectance is constant over

time and only the illumination

changes, can we solve for a single  
reflectance image and  $T$

illumination images  $\{L(x, y, t)\}_{t=1}^T$  ?

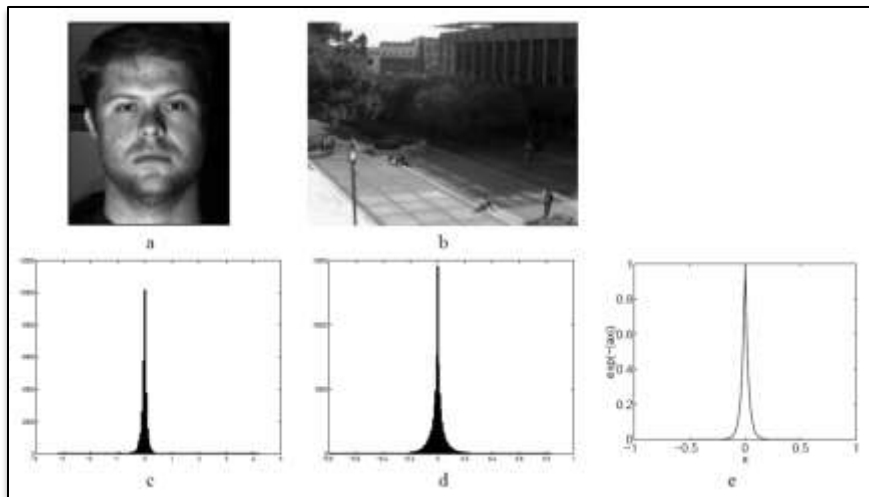
$$I(x, y) = L(x, y)R(x, y)$$



$$\{I(x, y, t)\}_{t=1}^T = \{L(x, y, t)\}_{t=1}^T R(x, y)$$

Still completely ill-posed : at every pixel there are  $T$   
equations and  $T+1$  unknowns.





- Prior based on intuition:
  - derivative-like filter outputs of  $L$  tend to be sparse

$$\{I(x, y, t)\}_{t=1}^T = \{L(x, y, t)\}_{t=1}^T R(x, y)$$



(move to log-space)

$$i(x, y, t) = r(x, y) + l(x, y, t)$$

$$o_n(x, y, t) = i(x, y, t) * f_n$$

$f_n$  = one of  $N$  filters like

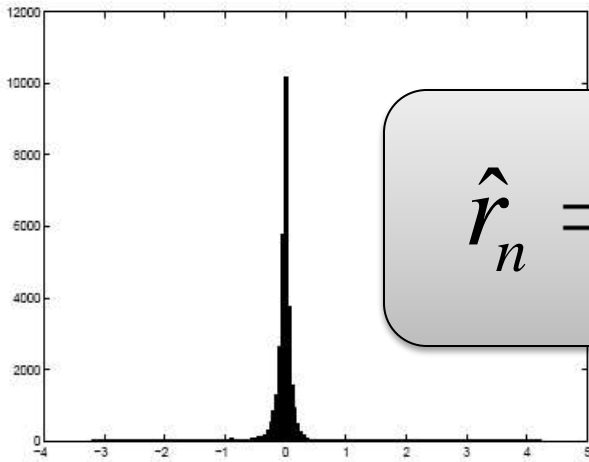
1	-1
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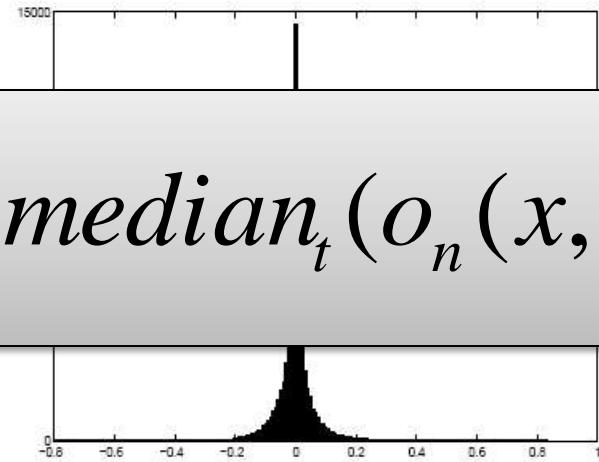
a



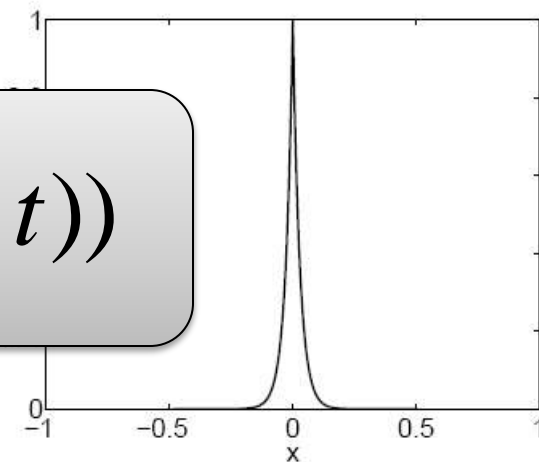
b



c



d



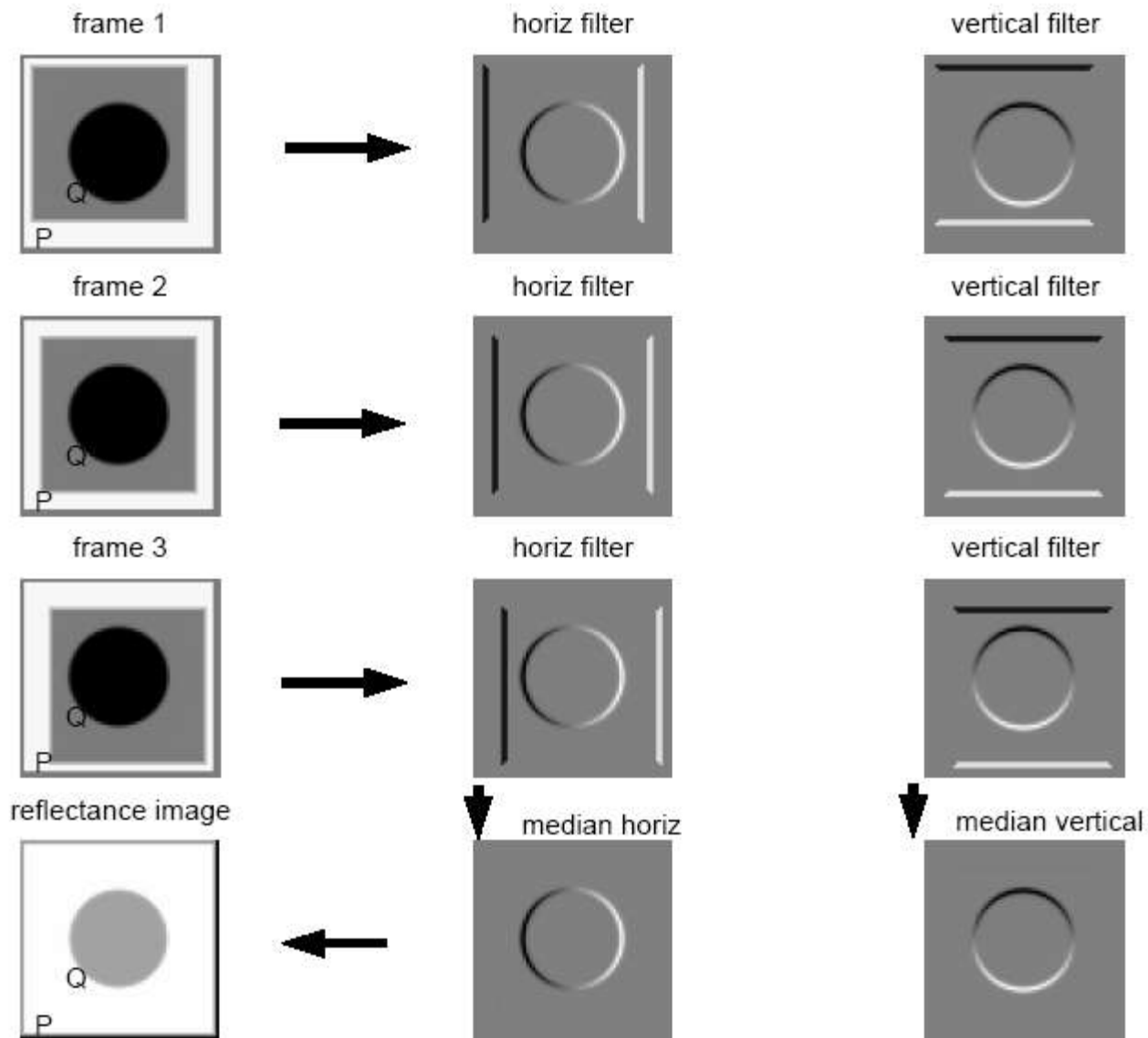
e

$$\hat{r}_n = \text{median}_t(o_n(x, y, t))$$

$$o_n(x, y, t) = i(x, y, t) * f_n$$

- Variety of responses has Laplacian-shaped distribution

# Toy Example

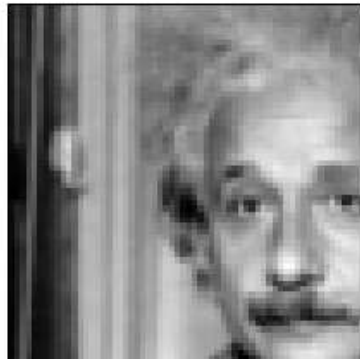


# Example Result 1

- Einstein image is translated diagonally  
4 pixels per frame



Reagan image



Einstein image



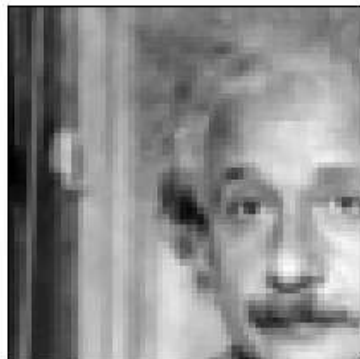
first frame



last frame



ML Reagan



ML Einstein



min filter



median filter

# Example Result 2

- 64 images with variable lighting from Yale Face Database



frame 2



frame 11



ML reflectance



ML illumination 2



ML illumination 11

# Intrinsic Images by Entropy Minimization

(Midway Presentation, by Yingda Chen)

Graham D. Finlayson, Mark S. Drew and Cheng Lu, ECCV,  
Prague, 2004

# Project Goals:

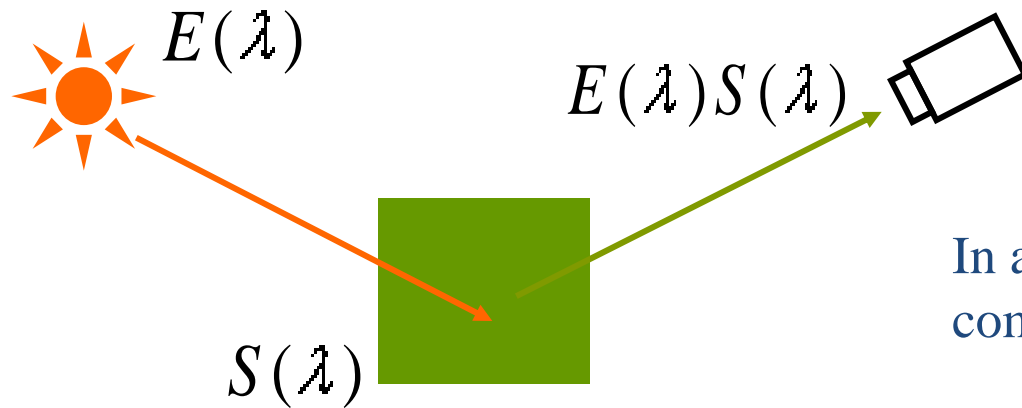
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Obtain the intrinsic image by removing shadows from images :

- Without camera calibration (no knowledge about the imagery source)
- Based on one single image (instead of multiple image arrays) by entropy minimization



# How an Image is Formed?



In an RGB image, the R, G , B components are obtained by:

$$r = \int R(\lambda)E(\lambda)S(\lambda)d\lambda$$

$$g = \int G(\lambda)E(\lambda)S(\lambda)d\lambda \quad (*)$$

$$b = \int B(\lambda)E(\lambda)S(\lambda)d\lambda$$

Camera responses depend on 3 factors:

- Light (E),
- Surface (S),
- Camera sensor (R,G, B)

# Planck's Law

- Blackbody:

A blackbody is a hypothetical object that *emits radiation at a maximum rate* for its given temperature and absorbs *all of the radiation* that strikes it.

Illumination sources such as radiator.



can be well approximately as a blackbody

- Planck's Law [Max Planck, 1901]

Planck's Law defines the energy **emission rate** of a blackbody , in unit of *watts per square meter per wavelength interval*, as a function of wavelength (in meters) and temperature  $T$  (in degrees Kelvin).

$$P_r(\lambda) = c_1 \lambda^{-5} \left( e^{\frac{c_2}{\lambda T}} - 1 \right)^{-1}$$

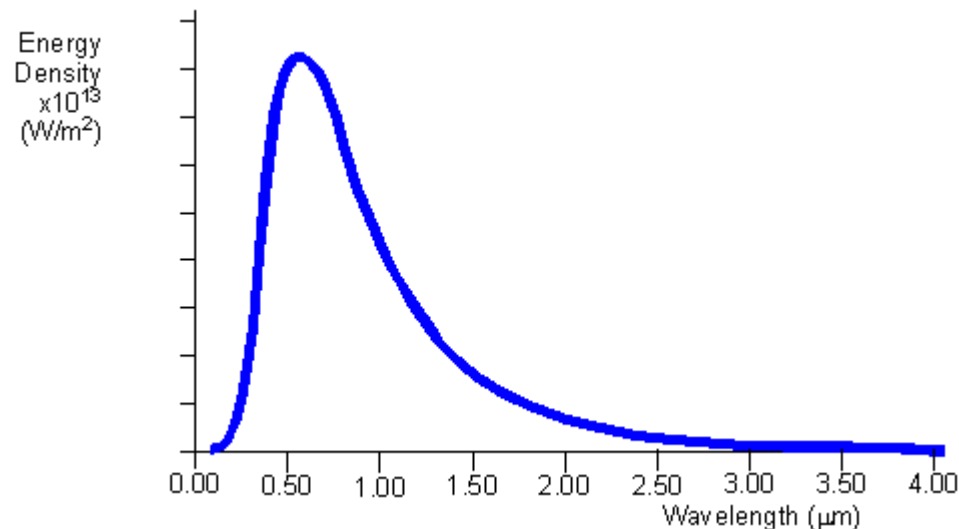
Where  $c_1 = 3.74183 \times 10^{-16} \text{W m}^2$  and  $c_2 = 1.4388 \times 10^{-2} \text{mK}$  are constants.

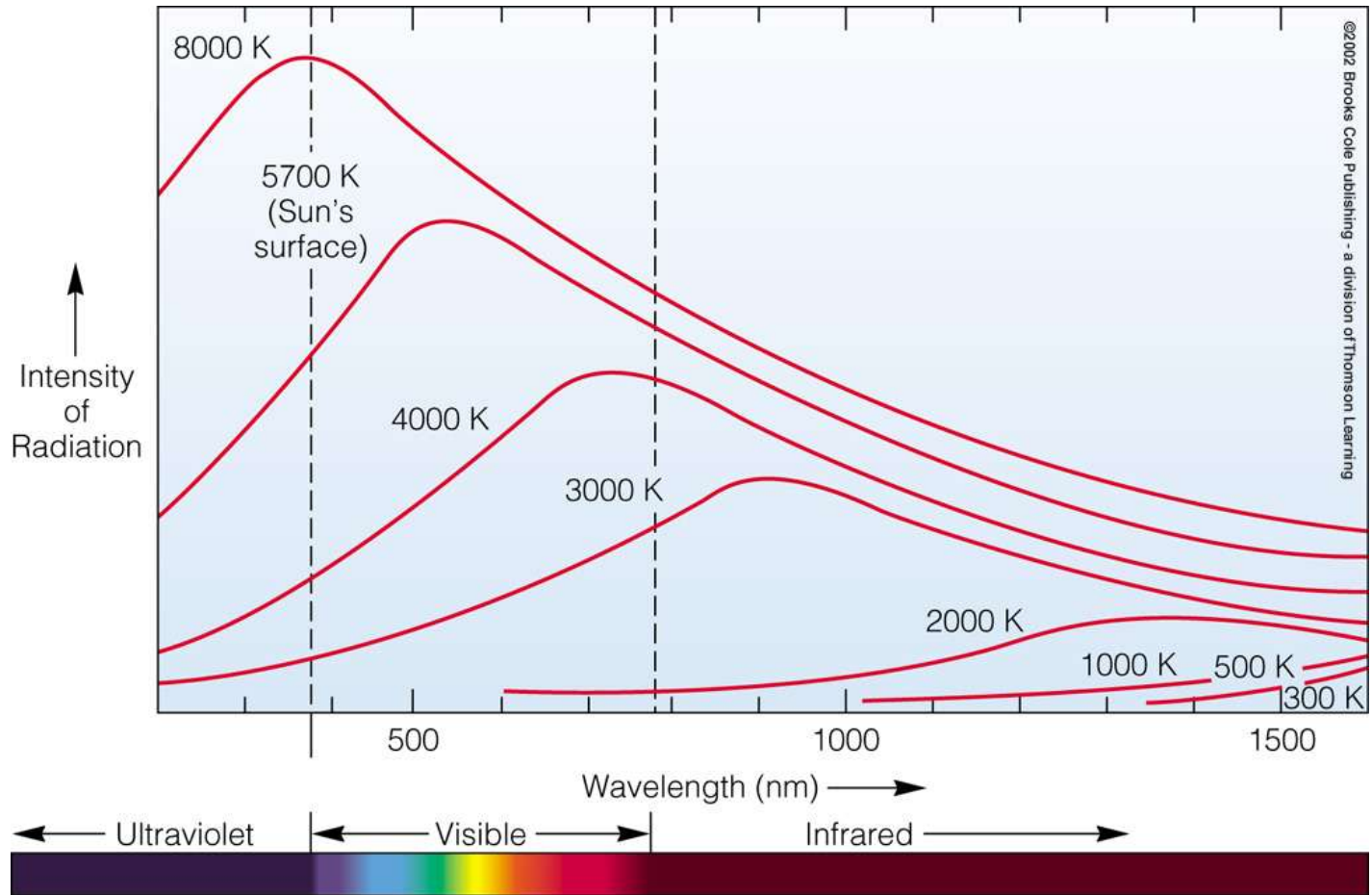
# Planck's Law (cont.)

Given the intensity of the radiation  $I$ , the Planck's law gives the spectral power of the lighting source:

$$E(\lambda) = I \times P_r = I c_1 \lambda^{-5} \left( e^{\frac{c_2}{\lambda T}} - 1 \right)^{-1} \cong I c_1 \lambda^{-5} e^{-\frac{c_2}{\lambda T}}$$

The temperature of a lighting source and the wavelength **together** determine the relative amounts radiation being emitted (color of the illuminator).





is **Blue**;

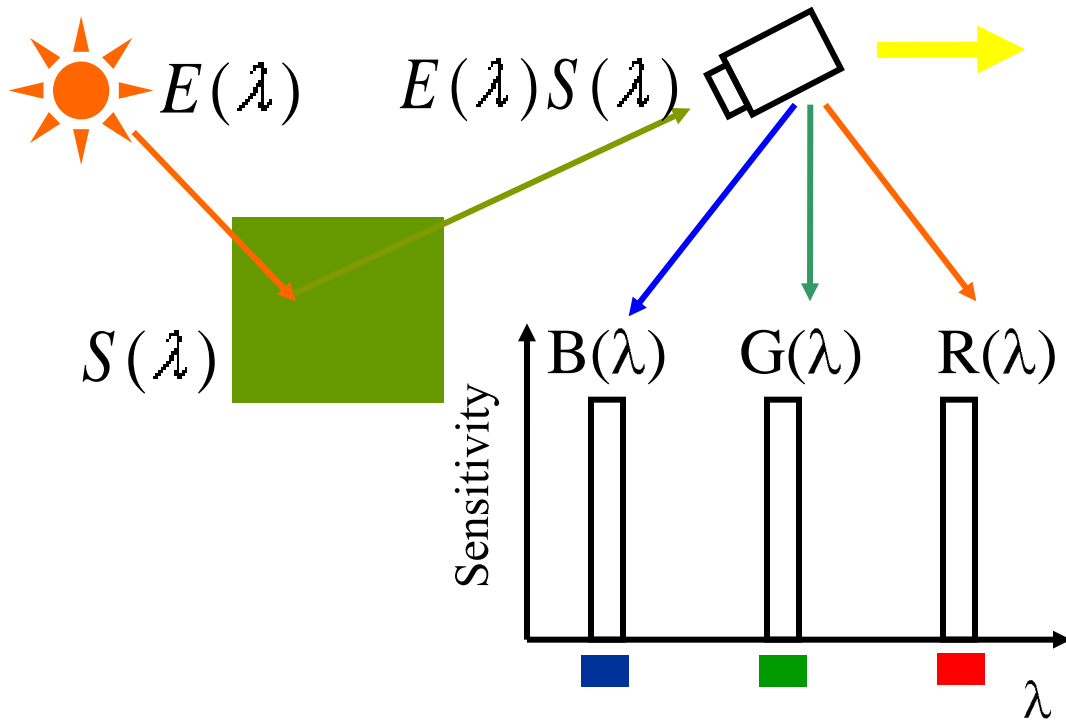


is **Red**;

# Image formation for Lambertian surface

Assume idea camera sensors:

$$R(\lambda) \equiv \delta(\lambda - \lambda_R) \quad G(\lambda) \equiv \delta(\lambda - \lambda_G) \quad B(\lambda) \equiv \delta(\lambda - \lambda_B)$$



$$\begin{aligned} r &= \int R(\lambda) E(\lambda) S(\lambda) d\lambda \\ &= \int \delta(\lambda - \lambda_R) E(\lambda) S(\lambda) d\lambda \\ &= E(\lambda_R) S(\lambda_R) \end{aligned}$$

...

$$g = E(\lambda_G) S(\lambda_G)$$

$$b = E(\lambda_B) S(\lambda_B)$$

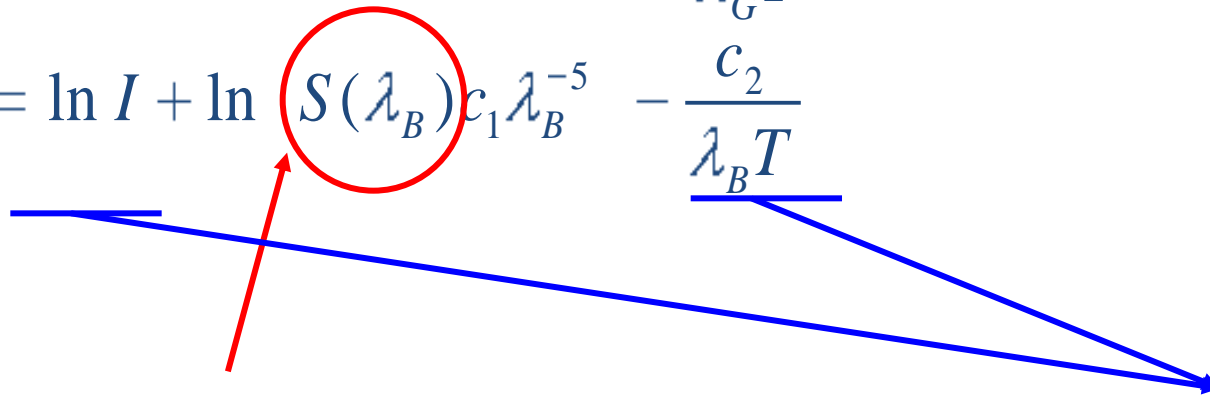
# Analysis of the components in image formation

$$\ln r = \ln I + \ln S(\lambda_R)c_1\lambda_R^{-5} - \frac{c_2}{\lambda_R T}$$

$$\ln g = \ln I + \ln S(\lambda_G)c_1\lambda_G^{-5} - \frac{c_2}{\lambda_G T}$$

$$\ln b = \ln I + \ln S(\lambda_B)c_1\lambda_B^{-5} - \frac{c_2}{\lambda_B T}$$

Need some manipulations to get rid of the illumination dependence



**Depends on the surface property only**

Depend on property of the illumination

# How to remove shadows (illumination)?

Define a 2-D chromaticity vector  $V$ ,

$$V = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} \ln(r / g) \\ \ln(b / g) \end{pmatrix}$$

$$v_1 = \ln \left( \frac{S(\lambda_R)\lambda_R^{-5}}{S(\lambda_G)\lambda_G^{-5}} \right) - \frac{c_2}{T} \left( \frac{\lambda_R}{\lambda_G} \right)$$

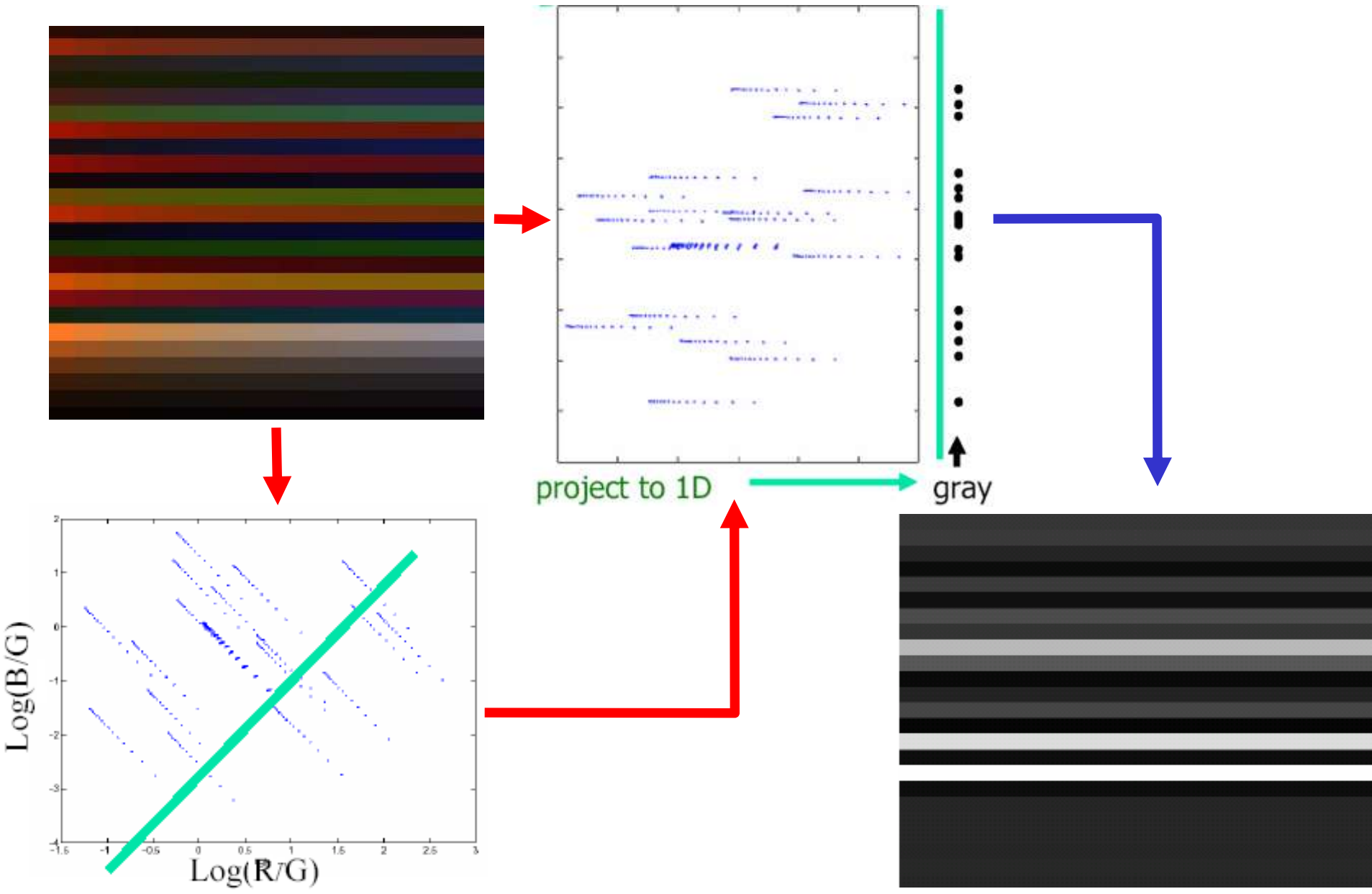
$$v_2 = \ln \left( \frac{S(\lambda_B)\lambda_B^{-5}}{S(\lambda_G)\lambda_G^{-5}} \right) - \frac{c_2}{T} \left( \frac{\lambda_B}{\lambda_G} \right)$$

The 2-D vector  $V$  forms a straight line in the space of *logs of ratios*, the slope of the which is determined by  $T$  (i.e. by illumination color). Project the 2D log ratios into the direction  $e^\perp$ , the 1-D grayscale invariant image can be obtained.  $e^\perp$  is the direction *orthogonal* to vector  $[\frac{c_2}{T} \left( \frac{\lambda_R}{\lambda_G} \right), \frac{c_2}{T} \left( \frac{\lambda_B}{\lambda_G} \right)]$

Shadow which occurs when there is a change in light but *not* surface will disappear in the invariant image



# How to remove shadows (illumination)? (cont.)



## How to remove shadows (illumination) ? (cont. II)

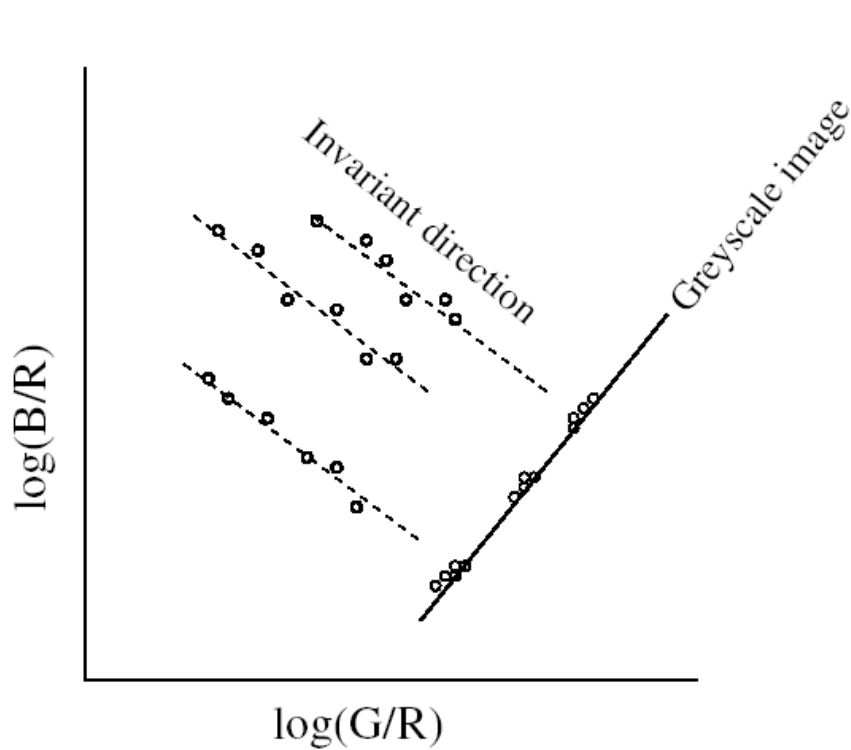
The key of the obtaining the invariant image is to determine the right projection direction. For a calibrated camera whose sensor sensitivity is known, the task is relatively easy.

- **Question:** *How to determine the projection direction for images from **Uncalibrated** Cameras?*

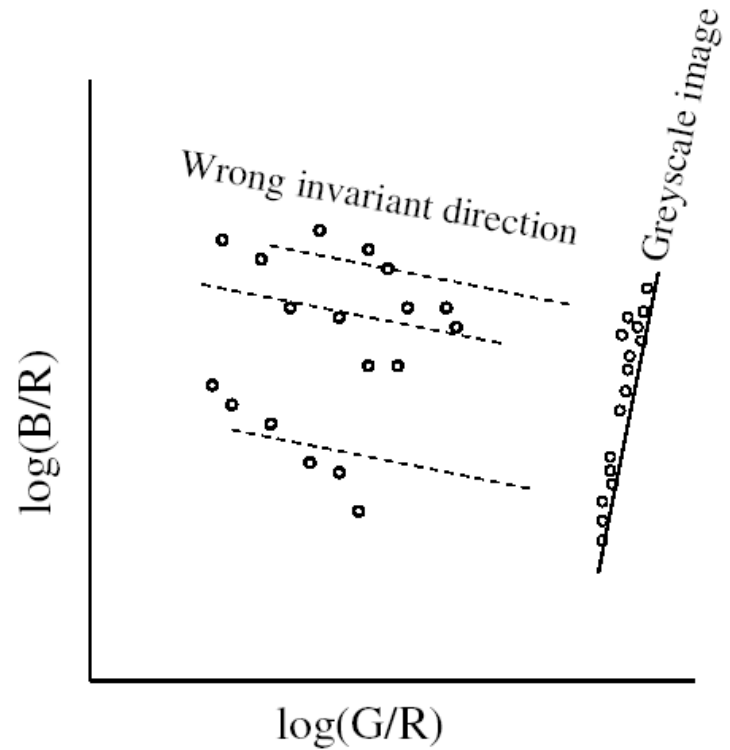
*Answer: Problematic, but artificial calibration can still be performed by obtaining a series of image from the same camera.*

- **Question:** *How to determine the projection direction for images whose source is unknown?*

# Entropy minimization



Correct Projection



Incorrect Projection

# Entropy minimization (cont.)

Given projection angle  $\theta$ , the projection result in a scalar value

$$\tau = \vec{V} \cdot [\cos \theta, \sin \theta]^T = v_1 \cos \theta + v_2 \sin \theta$$

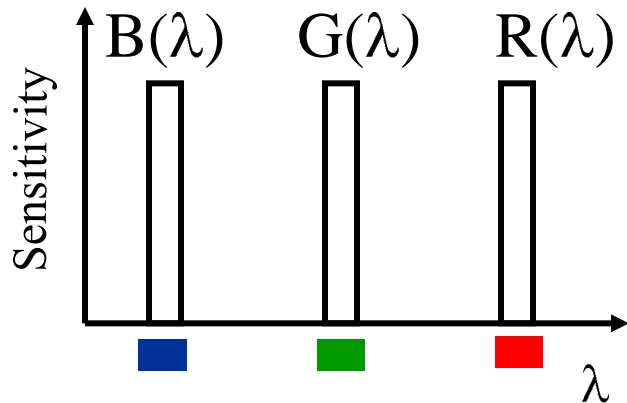
The scalar values can be encoded into a grayscale image, and the entropy be calculated as

$$H = - \sum_i p(x_i) \cdot \log p(x_i)$$

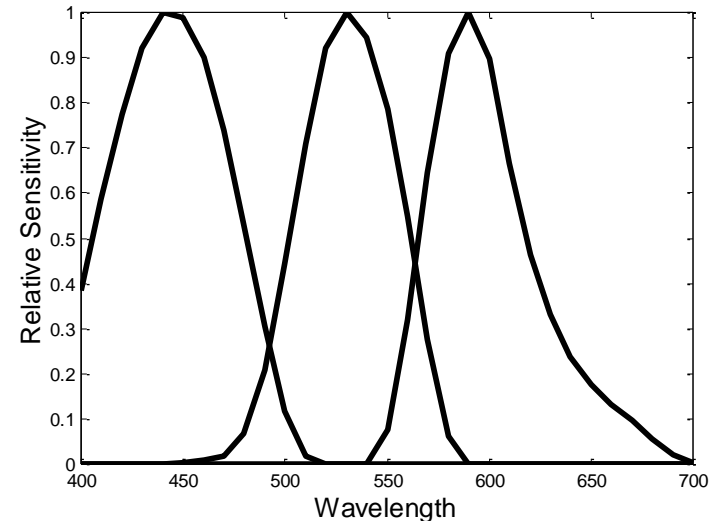
For each  $\theta = 1, 2, \dots, 180$ , we can obtain an corresponding entropy. As the more “spread-out” distribution results in a larger entropy value, ***the projection direction  $\theta$  that produces the minimum entropy is the correct projection direction***

# Assumptions?

- Delta sensor functions of camera



This assumption is idealized, but experiments show that it performs reasonably well.



- The image must be unbiased of R,G,B

This is NOT true for many images, which can be “reddish”, “bluish” or “greenish” in color. So some more dedicated approach should be introduced to remove (or at least suppress) the potential bias.

# Geometric Mean Invariant Image

Use the geometric mean as the reference color channel when taking the log ratios, so we will not favor for any particular color

$$C_{ref} = \sqrt[3]{r \times g \times b}$$

$$\ln(C_{ref}) = \ln(I) + \ln(c_1) + \frac{1}{3} \sum_{k=R,G,B} \ln(S(\lambda_k) \lambda_k^{-5}) - \frac{c_2}{T} \sum_{k=R,G,B} \frac{1}{\lambda_k}$$

$$\vec{\rho}' = \begin{pmatrix} \ln(r / C_{ref}) \\ \ln(g / C_{ref}) \\ \ln(b / C_{ref}) \end{pmatrix} = \begin{pmatrix} K + K_{11} - \frac{c_2}{T} K_{12} \\ K + K_{21} - \frac{c_2}{T} K_{22} \\ K + K_{31} - \frac{c_2}{T} K_{32} \end{pmatrix}$$

Where the K's are just constants

The geometric mean is unbiased, however, the invariant image is a projection from 2-D space into 1-D grayscale. The log ratios vector  $\vec{\rho}'$  is 3-D.

# Geometric Mean Invariant Image (cont.)

- From 3-D to 2-D before we can get the invariant.
- 

A 2 x 3  $U$  matrix can do this by  $\chi \equiv U \vec{\rho}'$

The transform should satisfy:

A straight line in the 3-D space is still straight after the transformation

- How do we find  $U$ ?

Since  $\rho_R + \rho_G + \rho_B = \ln(r \times g \times b / C_{ref}^3) = 0$

$$\Rightarrow \vec{\rho} \square \vec{u} = 0, \text{ where } \vec{u} = (1/\sqrt{3}) \square [1 \ 1 \ 1]^T$$

The orthogonal matrix  $U$  satisfies

$$U^T U = \mathbf{P}^\perp = I - \vec{u} \vec{u}^T$$

The rows of  $U$  are just the eigenvectors associated with the non-zero eigenvalues of the matrix  $\mathbf{P}^\perp$ .

# Geometric Mean Invariant Image (cont. II)

- With  $\chi \equiv U \vec{\rho}'$  we converted the 3-D log ratios space into 2-D, but with no color bias, the invariant image is then achieved by  $\tau = \chi \cdot [\cos \theta, \sin \theta]^T$ ,  $\theta$  is the correct projection angle, and we are back to the original track.

- Obtain  $\theta$  by entropy minimization:

- Decide the number of bins by Scott's rule:

$$\text{binNumber} = 3.5 \times \text{std}(\text{data}) \times N^{1/3}$$

- The probability of the  $i$ th bin is  $P_i = \frac{n_i}{N}$

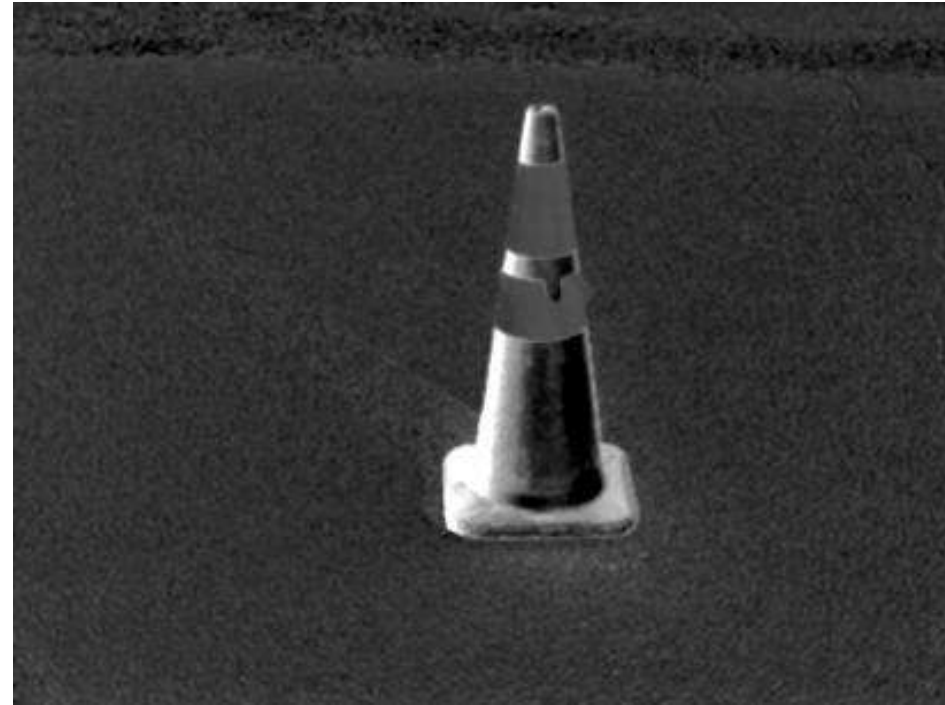
- The entropy is calculated as  $I = \sum_i -P_i \times \log_2(P_i)$



# Midway Results: invariant image

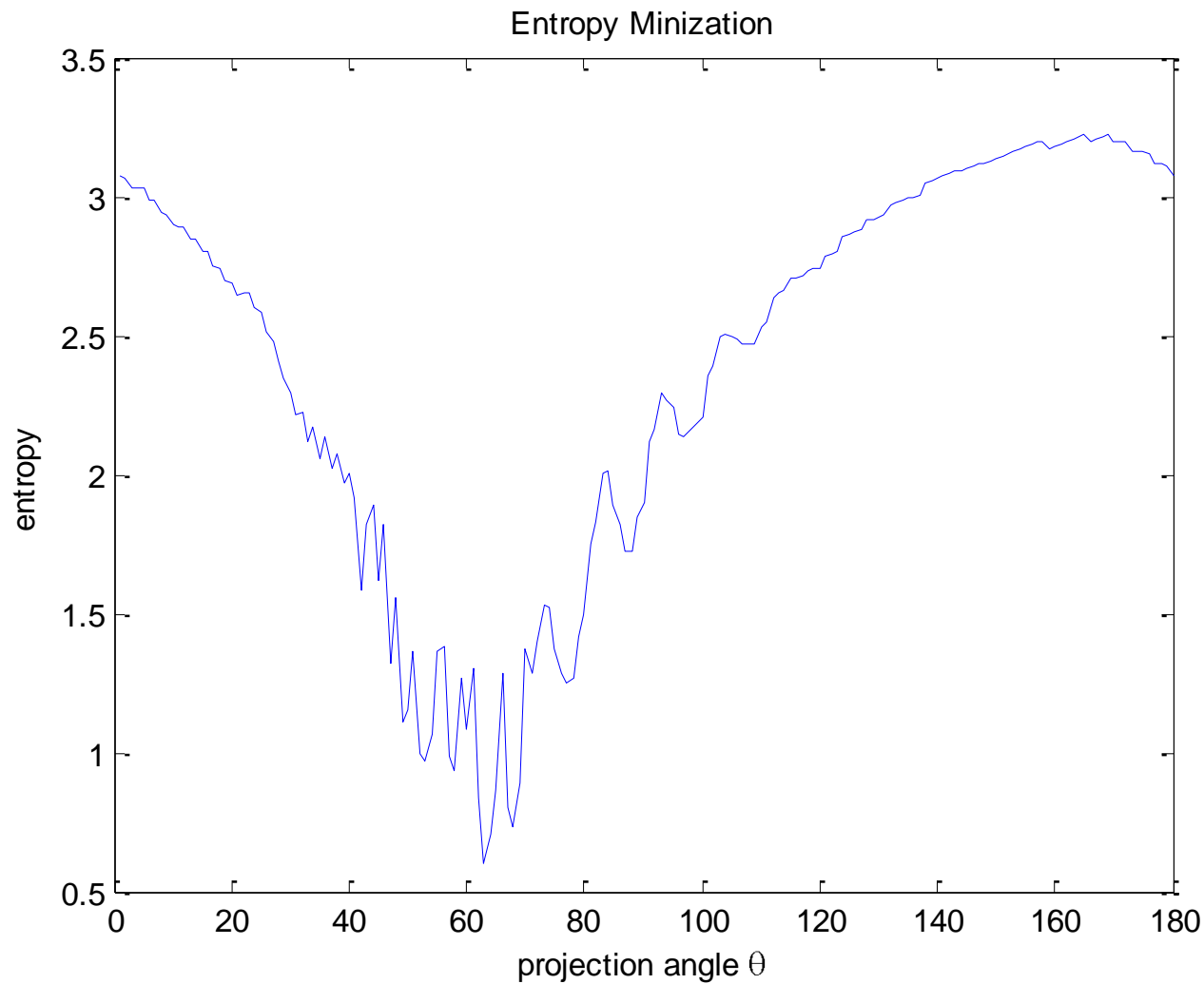


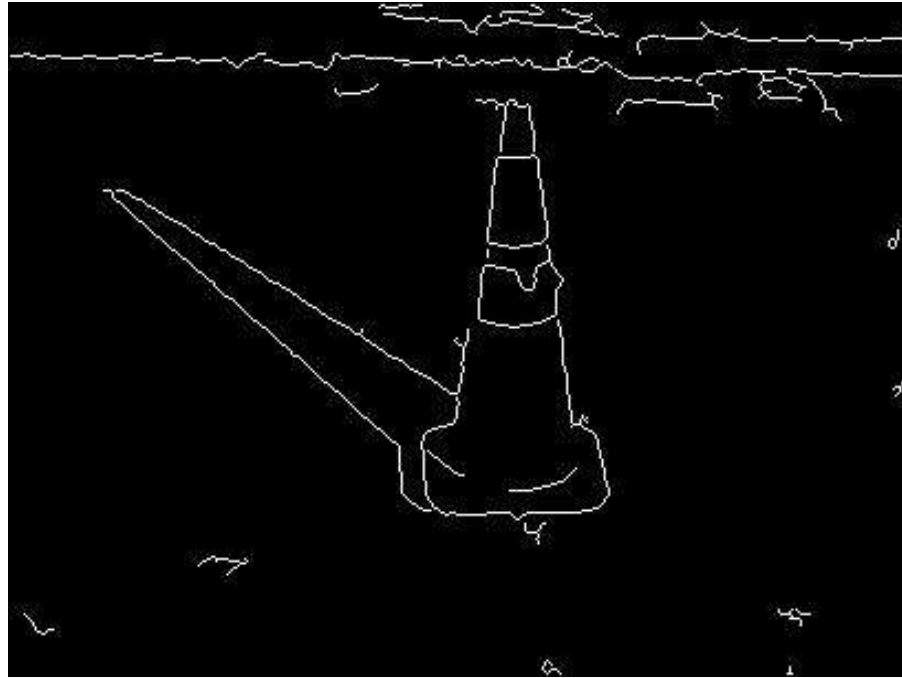
Original Image



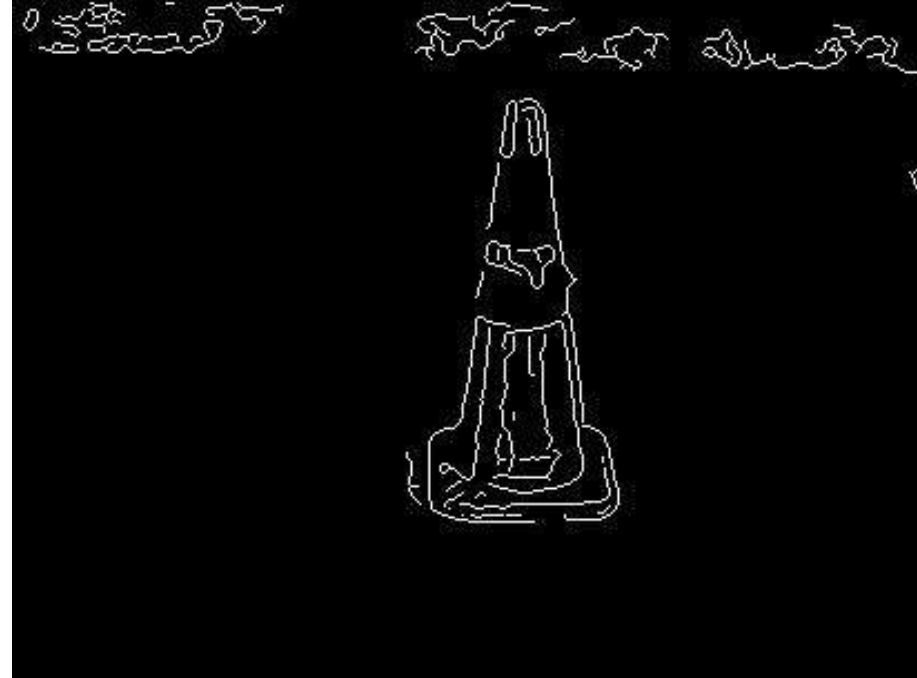
Invariant Image

# Entropy Minimization (Camera: Nikon CoolPix8700)





Edge in Original Image



Edge in Invariant Image

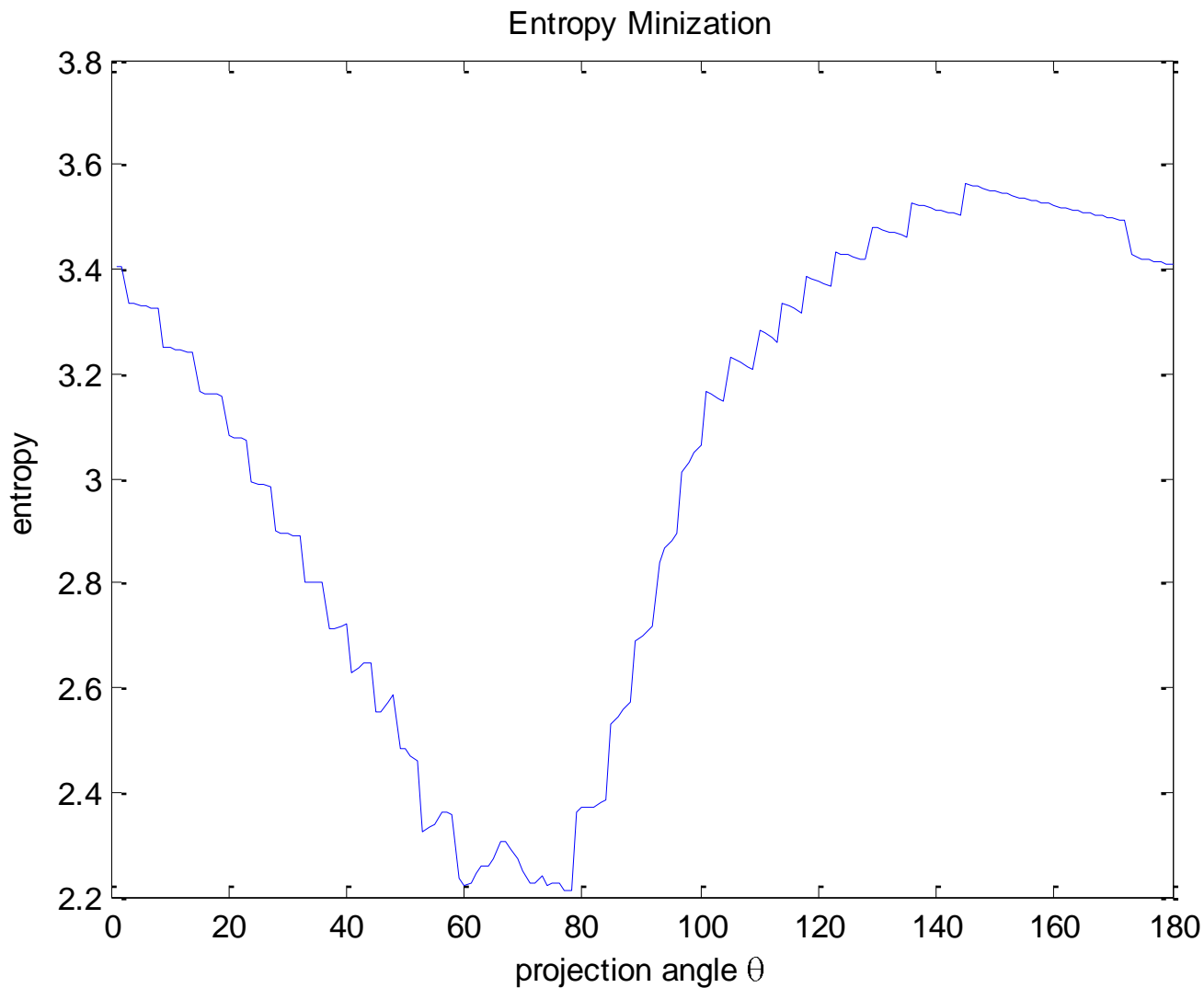


Original Image

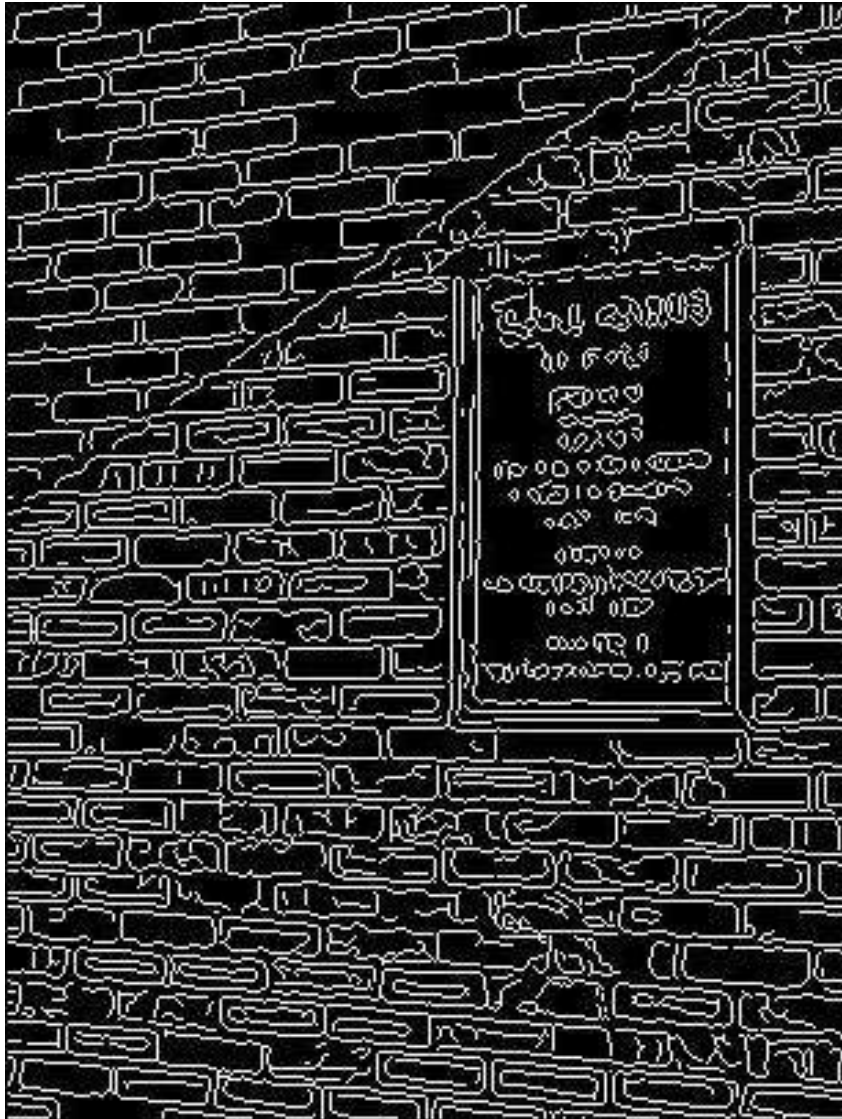


Invariant Image

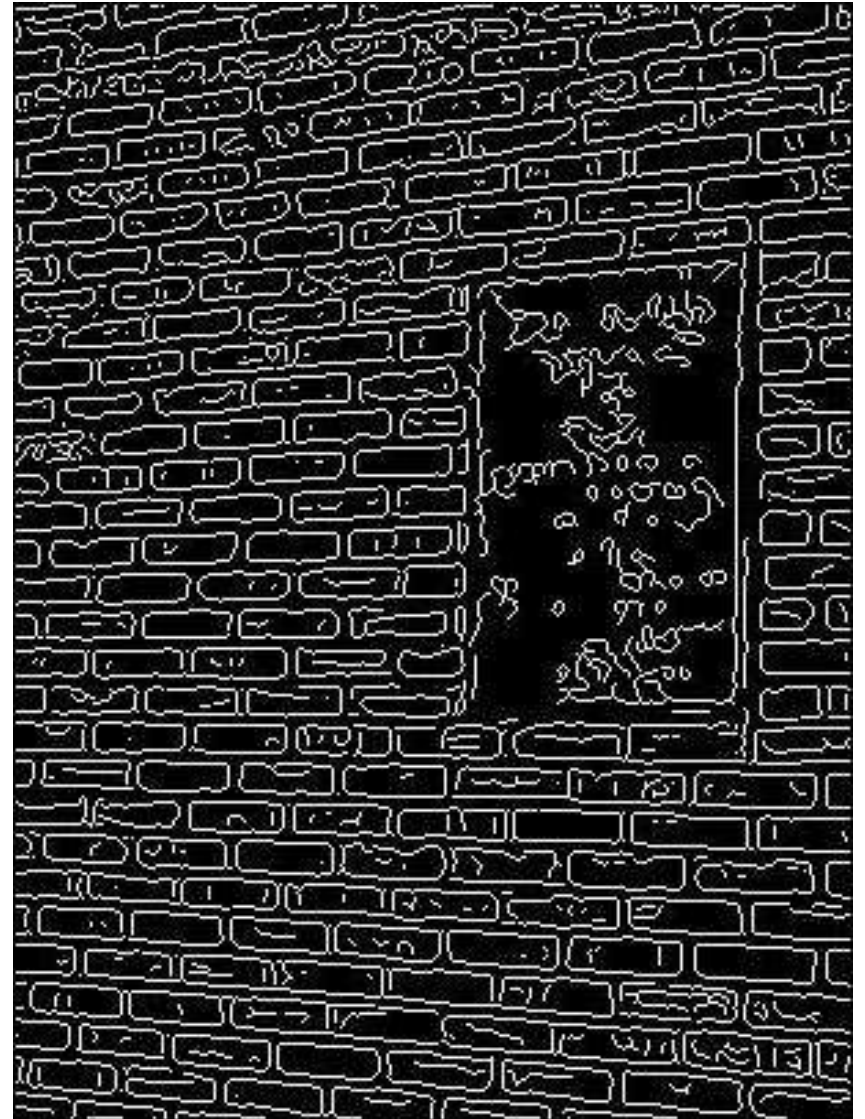
# Entropy Minimization (Camera: Nikon CoolPix8700)







Edge in Original Image



Edge in Invariant Image

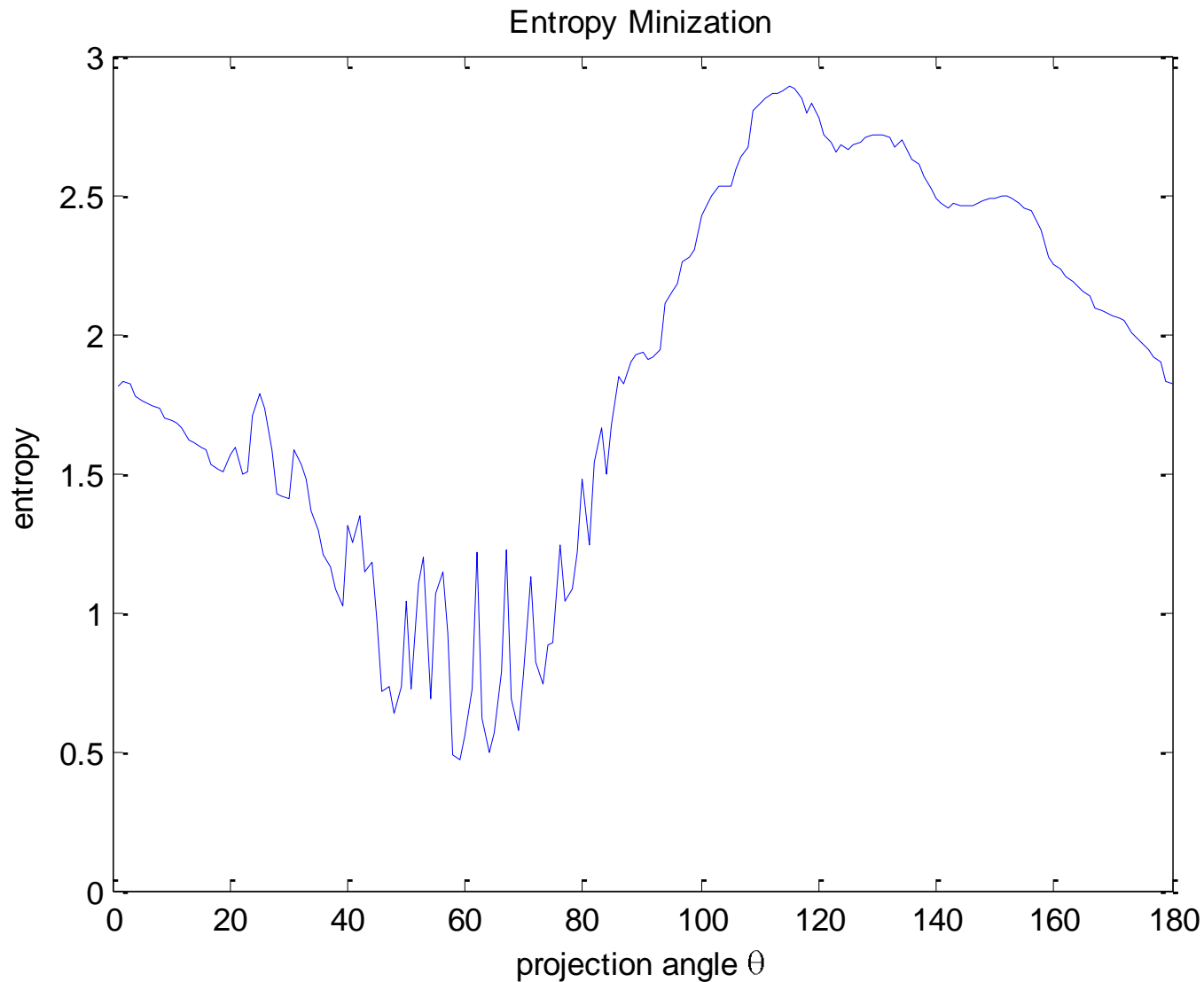


Original Image

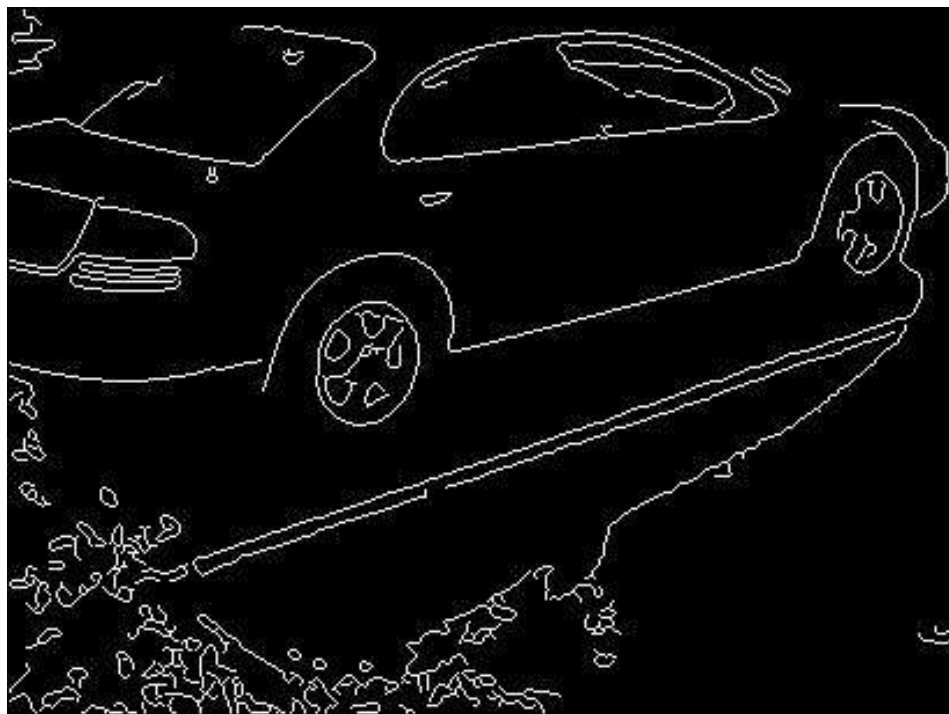


Invariant Image

# Entropy Minimization (Camera: Nikon CoolPix8700)







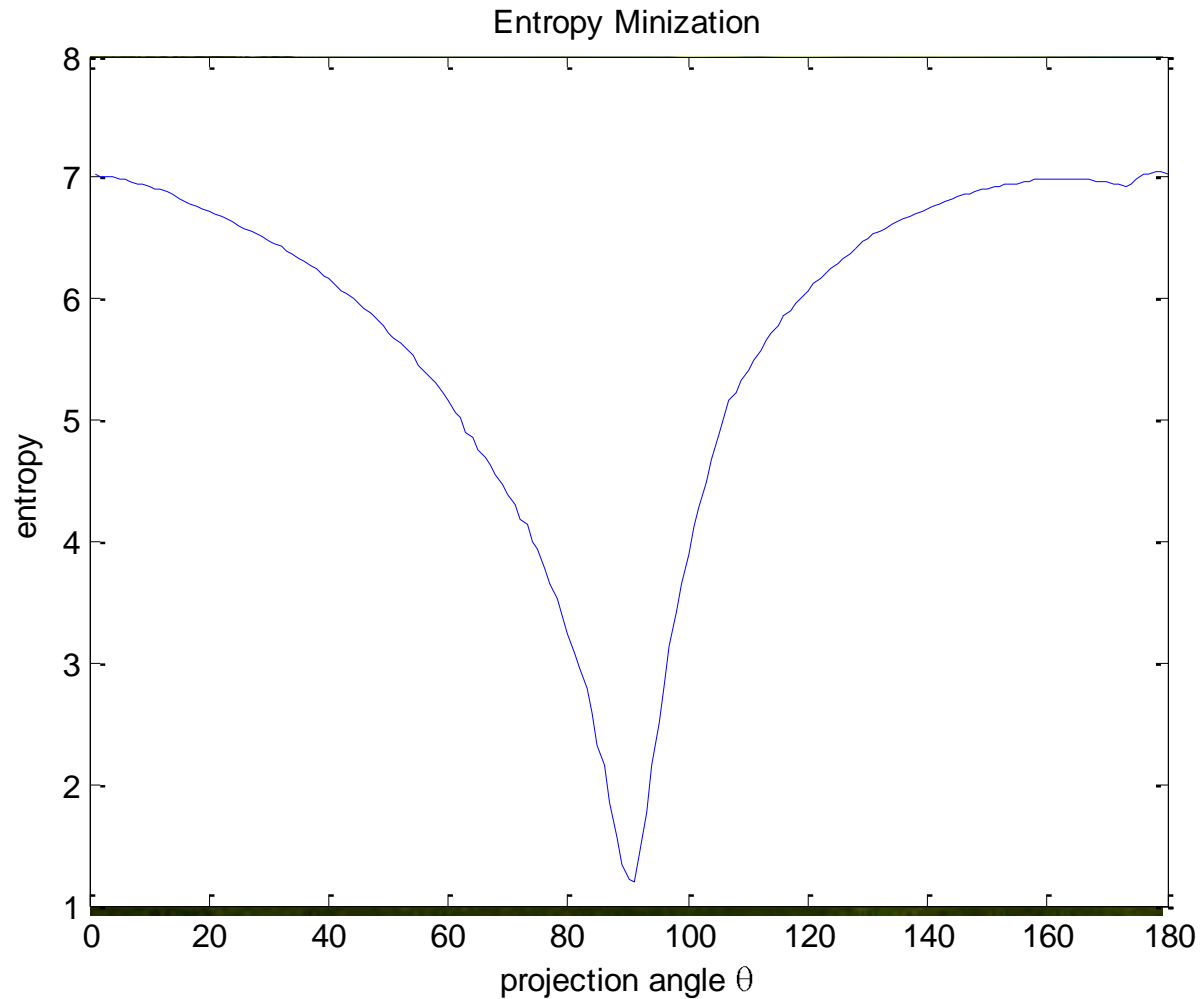
Edge in Original Image



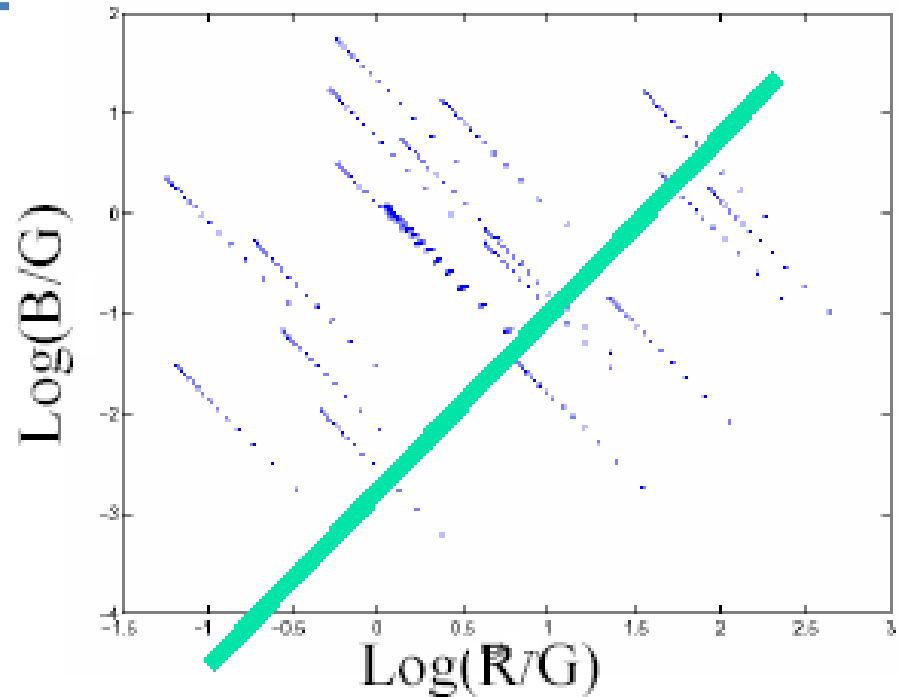
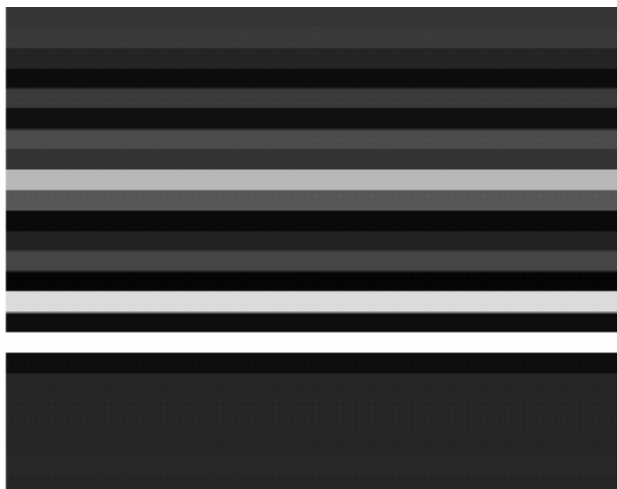
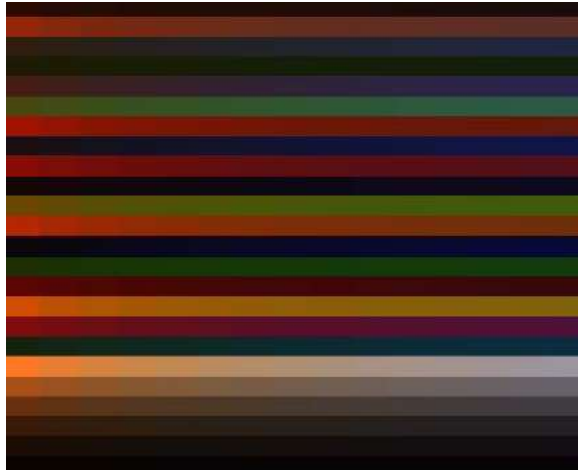
Edge in Invariant Image

# Entropy Minimization

(Camera: HP-912, better camera sensors?) [from author's website]



# Is the projected 1-D data really “colorless”?



We can recover 2-D chromaticity along the line

# Invariant chromaticity Image

- Recall that the grayscale image  $\tau = \chi \cdot e^\perp = \chi \cdot [\cos \theta, \sin \theta]^T$

let  $\chi_\theta = P_\theta \chi$  where  $P_\theta = e^\perp e^\perp{}^T$ , the 3-D log ratio is recovered by  $\rho = U^T \chi_\theta$

- Invariant chromaticity image:

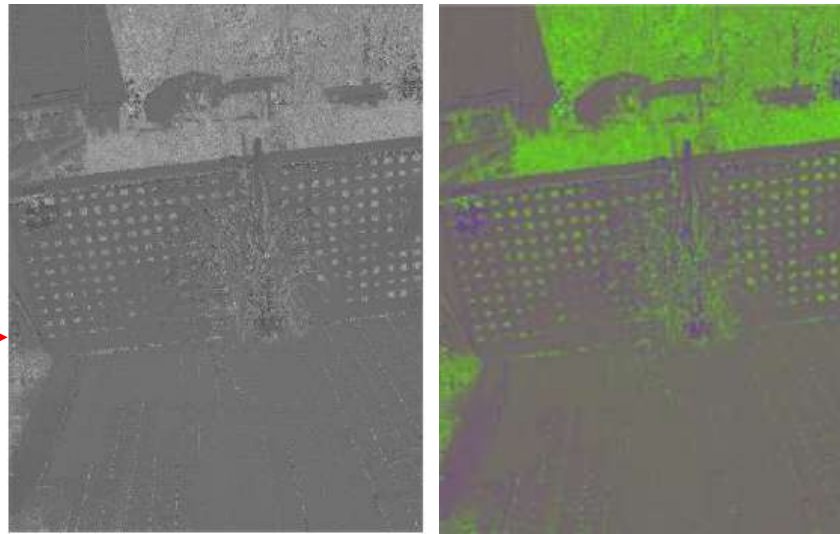
$$\tilde{r} = \exp(\rho)$$

$\tilde{r}$  is the invariant chromaticity image that presents the **color information inherent in the 1-D projection yet absent in the grayscale invariant image  $\tau$**

# Expected Results :



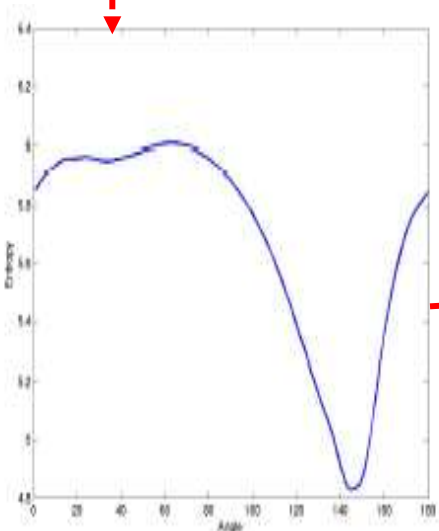
Original



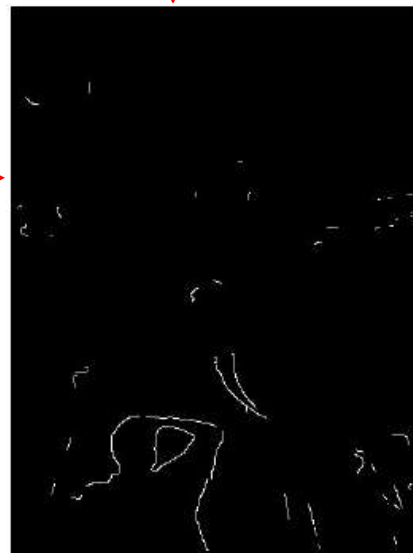
Invariant



Intrinsic image



Entropy Minimization

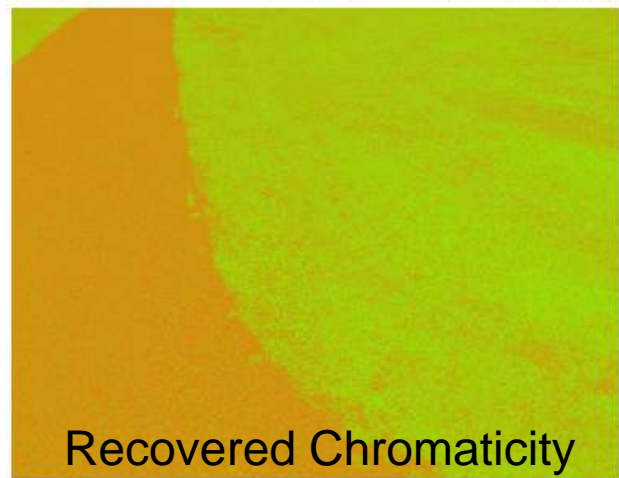


Shadow Edge

# Sweep Angle of Projection







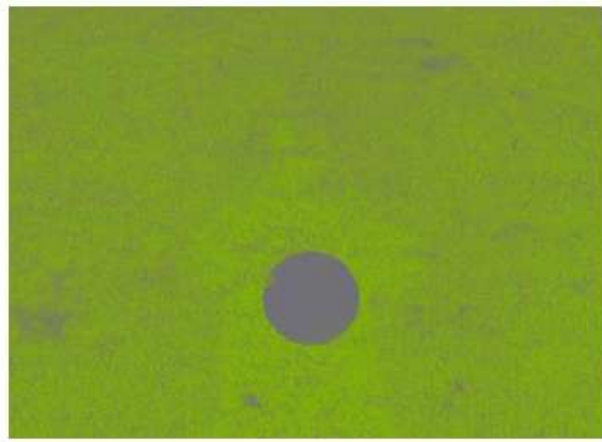
Recovered Chromaticity



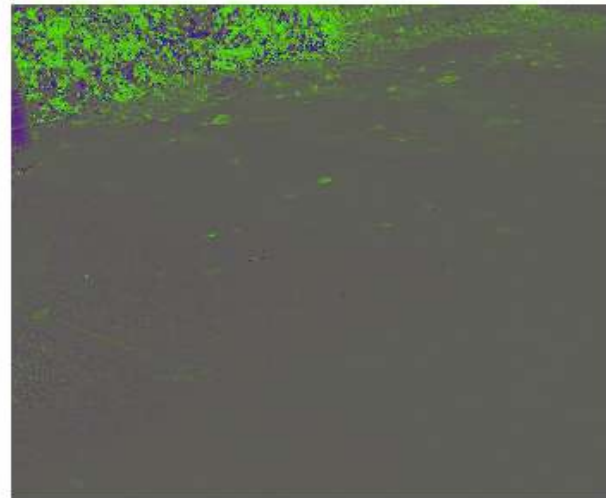














# Limitations of Shadow Removal

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- Only Hard shadows can be removed
- No overlapping of object and shadow boundaries
- Planckian light sources
- Narrow band cameras are idealized
- Reconstruction methods are texture-dumb